THEORY OF NONLINEAR VIBRATIONS IN A CLOSED PIPE

WITH A VIEW TO THERMOACOUSTIC EFFECTS

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The article presents a theory of gas vibrations in a closed pipe that makes it possible to calculate a finite amplitude in resonance vibrations. An axisymmetric case with a view to the compressibility and viscosity of the gas is examined.

Resonance vibrations of a gas column in pipes attracted and still attract the attention of researchers, partly because the nonsteady processes are the cause of intensified processes of heat and mass exchange [1].

Nonlinear vibrations in a pipe, with a harmonically moving piston at one end while the other end is firmly closed off, were the subject of numerous investigations, both in the USSR [2-4] and in other countries [5-10]; however, so far there is no theory that describes satisfactorily the properties of the phenomenon under examination.

In solving the problem, the following difficulties arise: a) with exact resonance $kL = \pi$ the vibration amplitude of the first harmonic becomes unbounded [10] which is physically senseless; b) the oscillations of pressure and speed are shifted in phase by $\pi/2$. This would indicate that the piston does not carry out work, whereas in experiments [10] thermoacoustic effects connected with heat transfer from the pipe to the environment were observed. This was observed even in vibrations at a frequency far from the resonance frequency.

In the present work we attempt to eliminate these disparities, i.e., we attempt to arrive at an acoustic approximation taking into account the finiteness of the amplitude of the vibrations of the first harmonic and the existence of thermoacoustic effects.

We examine the movement of a viscous compressible liquid with constant physical properties in a long cylindrical pipe ($R/L \ll 1$). We introduce the small parameter $\varepsilon = U_\infty/(\omega L)$ and represent the physical magnitudes in the form of an expansion into a power series of ε. The solution of the equations in the first approximation with high-frequency vibrations $R\sqrt{\omega/2\nu} \gg 1$ has the form

$$p_1 = -(A\exp(ikx) + B\exp(-ikx))\exp(i\omega t),$$

$$u_1 = \frac{1}{\rho_0 c_0}(A\exp(ikx) - B\exp(-ikx))\left[1 - \sqrt{\frac{R}{r}}\exp(-(1+i)\sqrt{\omega/2\nu}(R-r))\right]\exp(i\omega t),$$

$$v_1 = \frac{(1+i)\omega}{2\rho_0 c_0^2}\sqrt{\frac{2\nu}{\omega}}\left\{\left(\frac{\varkappa-1}{\sqrt{\Pr}}+1\right)\frac{R}{r} - \sqrt{\frac{R}{r}}\times\right.$$

$$\left.\left[\exp(-(1+i)\sqrt{\omega/2\nu}(R-r)) + \frac{\varkappa-1}{\sqrt{\Pr}}\exp(-(1+i)\sqrt{\omega\Pr/2\nu}(R-r))\right]\right\}p_1, \qquad (1)$$

$$\rho_1 = \frac{1}{c_0^2}\left[1 + (\varkappa-1)\sqrt{\frac{R}{r}}\exp(-(1+i)\sqrt{\omega\Pr/2\nu}(R-r))\right]p_1,$$

$$T_1 = \frac{1}{\rho_0 c_p}\left[1 - \sqrt{\frac{R}{r}}\exp(-(1+i)\sqrt{\omega\Pr/2\nu}(R-r))\right]p_1,$$

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where the number in the subscript denotes the corresponding approximation. We point out that expressions (1) differ from [10] firstly by the fact that in all equations it was taken that $R\sqrt{\omega/2\nu} \gg 1$, and in view of the closeness of Pr to unity $R\sqrt{\omega \mathrm{Pr}/2\nu} \gg 1$; secondly, the third expression is written taking into account that at the wall $v_1 \equiv 0$.

For determining the constants A and B in [10], the following conditions were used:

$$u_1(x=0,\ r=0) = \omega l \exp(i\omega t),\ u_1(x=L,\ r=0) = 0. \tag{2}$$

In order to simplify the analysis, we put

$$A = -\frac{|C|}{2}\exp(i\alpha),\ B = -\frac{|C|}{2}\exp(-i\alpha), \tag{3}$$

where $|\ |$ indicates that we take the absolute value. Then we obtain

$$p_1 = |C|\cos(kx+\alpha)\exp(i\omega t),\ u_1 = -\frac{i|C|}{\rho_0 c_0}\sin(kx+\alpha)\left[1-\sqrt{\frac{R}{r}}\exp(-(1+i)\sqrt{\omega/2\nu}(R-r))\right]\exp(i\omega t). \tag{4}$$

We assume that α is a complex magnitude, i.e., $\alpha = \alpha_0 + i\beta$, and we take into account that in the process of propagation the wave undergoes absorption. This may be done if it is assumed that the wave number is also complex, i.e., we adopt [11] $k = k_0(1 + i\eta)$. Furthermore, we write the first of the conditions (2) in the form

$$u_1(x=0,\ r=0) = \omega l(\cos\varphi + i\sin\varphi)\exp(i\omega t), \tag{5}$$

and the second condition remains without change. Then for determining the five unknowns $|C|$, φ, η, α_0, and β, we obtain the following system of four equations:

$$\begin{gathered}\omega l\cos\varphi = -\frac{|C|}{\rho_0 c_0}\cos\alpha_0\,\mathrm{sh}\,\beta,\ \omega l\sin\varphi = -\frac{|C|}{\rho_0 c_0}\sin\alpha_0\,\mathrm{ch}\,\beta,\\ \alpha_0 + k_0 L = 0,\ \beta + k_0\eta L = 0.\end{gathered} \tag{6}$$

With (6) taken into account, expression (4) assumes the form

$$p_1 = |C|\cos[k_0(x-L) + i\beta(1-x/L)]\exp(i\omega t), \tag{7}$$

$$u_1 = -\frac{i|C|}{\rho_0 c_0}\sin[k_0(x-L) + i\beta(1-x/L)]\left[1-\sqrt{\frac{R}{r}}\exp(-(1+i)\sqrt{\omega/2\nu}(R-r))\right]\exp(i\omega t).$$

We can obtain the lacking condition on the basis of the following consideration: The amount of energy averaged over time, supplied by the piston to the pipe, has to be equal to the amount of heat transmitted through the pipe walls to the environment.

The work of the piston can be easily calculated: We average the product of pressure by speed on the piston and integrate over the cross-sectional area of the pipe

$$W_1 = \pi R^2\langle p_1(x=0)\cdot u_1(x=0,\ r=0)\rangle, \tag{8}$$

where $\langle\ \rangle$ denotes averaging in time. Expression (8) takes into account the fact that with $R\sqrt{\omega/2\nu} \gg 1$ the flow rate may be considered to be equal to the speed in the core of the flow.

It is easy to show that if two complex functions Φ and Ψ change in time harmonically, their product averaged in time can be calculated by the ratio

$$\langle\Phi\cdot\Psi\rangle = \frac{m(\alpha)n^*(\alpha) + m^*(\alpha)n(\alpha)}{4}, \tag{9}$$

where $m(\alpha)$, $n(\alpha)$ are the complex amplitudes of the functions Φ and Ψ, the asterisk denotes that the complex conjugate value is taken. With a view to (9) and after some transformations we obtain

$$W_1 = \frac{\pi R^2}{4\rho_0 c_0}|C|^2\,\mathrm{sh}\,2\beta. \tag{10}$$

To determine the heat fluxes due to thermoacoustic effects, we examine the equation of the second approximation for the temperature field [10]:

$$\rho_0 c_p \frac{\partial T_2}{\partial t} - \frac{\partial p_2}{\partial t} - \frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right) = -\rho_0 c_p\left[u_1\frac{\partial T_1}{\partial x} + v_1\frac{\partial T_1}{\partial r}\right] - \rho_1 c_p\frac{\partial T_1}{\partial t} + \mu\left(\frac{\partial u_1}{\partial r}\right)^2 + u_1\frac{\partial p_1}{\partial x} \quad (11)$$

with the boundary conditions

$$\frac{\partial T_2}{\partial r} = 0,\ r = 0;\ T_2 = 0,\ r = R. \quad (12)$$

Since the magnitudes of second order contain a part averaged in time and a part pulsating with double frequency [1], and in our case the averaged part is of interest, we average Eq. (11) in time and obtain

$$\left\langle -\frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right)\right\rangle = -\rho_0 c_p\left\langle u_1\frac{\partial T_1}{\partial x} + v_1\frac{\partial T_1}{\partial r}\right\rangle - c_p\left\langle \rho_1\frac{\partial T_1}{\partial t}\right\rangle + \mu\left\langle\frac{\partial u_1}{\partial r}\right\rangle^2 + \left\langle u_1\frac{\partial p_1}{\partial x}\right\rangle. \quad (13)$$

In this last expression it is taken that the process is already steady, otherwise the term $\langle \partial T_2/\partial t \rangle$ would be nonzero. We multiply (13) by r and integrate from 0 to R. Then, with a view to the boundary conditions and after a number of transformations we find

$$\langle q \rangle = -\left\langle \lambda \frac{\partial T_2}{\partial r}\Big|_R\right\rangle = -\frac{c_p\rho_0}{R}\left[\int_0^R r\frac{\partial\langle u_1T_1\rangle}{\partial x}dr + \int_0^R \frac{\partial\langle rv_1T_1\rangle}{\partial r}dr\right]. \quad (14)$$

If we substitute the expressions u_1, T_1, and v_1 from (1) into (14) and carry out the necessary calculations, we obtain for the local heat flux

$$\langle q \rangle = \frac{1}{8}\rho_0 c_0^2|\bar{C}|^2\sqrt{2\nu\omega}\left\{\left[\frac{2(1-\sqrt{\mathrm{Pr}})}{1+\mathrm{Pr}} + \frac{\varkappa+1}{\sqrt{\mathrm{Pr}}} - 1\right]\times \right.$$
$$\left.\times\cos 2k_0L(1-x/L) + \left[1 + \frac{\varkappa-1}{\sqrt{\mathrm{Pr}}}\right]\mathrm{ch}\,2\beta\,(1-x/L)\right\}, \quad (15)$$

where $|\bar{C}| = |C|/(\rho_0 c_0^2)$.

The heat losses from the entire surface of the pipe are found by integration of the expression over the lateral surface of the pipe:

$$W_2 = -\frac{1}{4}\pi RL\rho_0c_0^2|\bar{C}|^2\sqrt{2\nu\omega}\left\{D\frac{\sin 2k_0L}{2k_0L} + H\frac{\mathrm{sh}\,2\beta}{2\beta}\right\}, \quad (16)$$

where $D = \frac{2(1-\sqrt{\mathrm{Pr}})}{1+\mathrm{Pr}} + \frac{\varkappa+1}{\sqrt{\mathrm{Pr}}} - 1;\ H = 1 + \frac{\varkappa-1}{\sqrt{\mathrm{Pr}}}$.

Taking into account that $W_1 + W_2 = 0$, we may obtain for determining β that

$$(2\beta R/\delta - Hk_0L)\,\mathrm{sh}\,2\beta = \beta D\sin 2k_0L, \quad (17)$$

where $\delta = \sqrt{2\nu/\omega}$, and in view of relation (6),

$$|\bar{C}| = \frac{k_0L}{\sqrt{\cos^2 k_0L\,\mathrm{sh}^2\beta + \sin^2 k_0L\,\mathrm{ch}^2\beta}}\,\frac{l}{L}. \quad (18)$$

For $\beta \to 0$, $\cosh\beta \to 1$, $\sinh\beta \to 0$, we have $|\bar{C}| = \frac{k_0 l}{\sin k_0L}$, i.e., we obtain a result similar to [10], which is correct only for frequencies far from resonance. The most important consequence of the suggested theory is that the result is bounded even with exact resonance: $|\bar{C}| = k_0 l/\sinh\beta$.

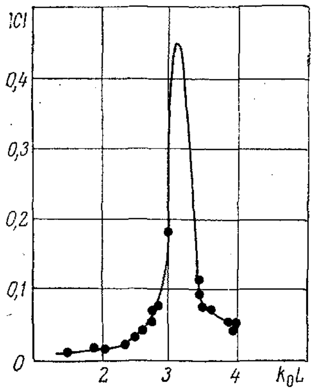

Fig. 1. Resonance curve of the dependence of the vibration amplitude on the frequency (curve: theory; dots: experimental data of [10]).

Figure 1 shows the resonance curve according to expressions (17), (18) and the corresponding experimental points from [10] for L = 1.7 m, l = 0.0138 m, R = 0.0095 m. The results agree well with each other.

An important characteristic of the process of wave propagation with finite amplitude is the spatial attenuation factor [12] which can be easily obtained from the fourth of Eqs. (6) if it is taken into account that $-k_0\eta = \zeta$. Putting $\sinh 2\beta \sim 2\beta$ for small β, we have from (17):

$$\zeta = \frac{\beta}{L} = (\delta/4RL)(D \sin 2k_0L + 2Hk_0L). \tag{19}$$

Calculations by (19) show that ζ turns out to be 2-3 times smaller than was found by experiments carried out with sawtooth waves [12], but the change of ζ over the frequency coincides qualitatively.

We also want to point out that with increasing vibration amplitude, the contribution of the terms of higher orders to the thermoacoustic effects increases. In our work we took into account only terms of second order. An evaluation shows that even in exact resonance the contribution of terms of fourth order does not exceed 30%. If such an accuracy suffices, then the simplicity of the obtained expressions justifies neglecting terms of higher orders. Taking these terms into account if the vibration amplitude has to be determined more accurately does not present any fundamental difficulties on the basis of the suggested method.

NOTATION

k, wave number; L, length of the pipe; R, radius of the pipe; ε, small parameter; U_∞, maximum amplitude of speed pulsations; ω, cyclic vibration frequency; ν, kinematic viscosity; p, pressure; ρ, density of the gas; T, gas temperature; u, v, axial and radial speed components, respectively; $\varkappa = c_p/c_V$, ratio of specific heats; x, r, axial and radial coordinates, respectively; Pr, Prandtl number; A, B, and C, α, complex constants determined by the boundary conditions; c_0, speed of sound in the unperturbed medium; ρ_0, density of the unperturbed medium; l, piston stroke; α_0, β, real and imaginary parts of α, respectively; k_0, real part of the wave number; η, coefficient taking absorption into account; φ, angle; W_1, work of the piston; W_2, heat losses; Φ, Ψ, complex functions with amplitudes $m(\alpha)$ and $n(\alpha)$, respectively; μ, coefficient of dynamic viscosity; δ, thickness of the acoustic boundary layer; D, H, coefficients depending on $\varkappa$ and Pr, respectively; ζ, spatial attenuation factor.

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NONSTEADY HEAT AND MASS TRANSFER IN DRYING BY REDUCED PRESSURE

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The article examines nonsteady heat and mass transfer in the process of drying by reduced pressure, on the assumption that the moisture is situated on the surface of an infinite plate.

Drying by reduced pressure may be effected in various regimes: linear reduction of moisture content, linear reduction of temperatures, or constant rate of reducing pressure. The most favorable regime is linear reduction of moisture content which makes it possible to carry out the process of moisture removal at optimum speed. In this case the moisture content has to change according to the regularity

$$U = U_{\text{i}} - N\tau. \tag{1}$$

The drying rate is determined on the basis of the technological requirements that the product has to fulfill. The drying rate depends on the heat flux supplied for evaporating the liquid [1]:

$$N = \frac{qf}{rm_{\text{d.m.}}}. \tag{2}$$

If we neglect the change of evaporation heat, of the heat-exchange surface, and of the weight of dry substance in the drying process, then the constant value of the drying rate is determined by the constant value of the heat flux. In the process of realizing this regime of drying by reduced pressure, evaporation of moisture is effected by liberation of the internal energy of the moist material [2].

If there is considerable thermal resistance or if the particles are large, the evaporation process of the moisture will to a certain extent be affected by the inhomogeneity of the temperature field inside the particles. Finding the temperature field is connected with the solution of the differential equation of heat conduction. For an infinite plate, on condition that the moisture is only on the surface, the equation is as follows:

$$\frac{\partial T(x;\ \tau)}{\partial \tau} = a\,\frac{\partial^2 T(x;\ \tau)}{\partial x^2}. \tag{3}$$

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